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GRADIENT CYBERNETICS

The Calculus of Recursion

SYNTHESIS ESSAY VI

The Helical Worldline on S²

The Exact Derivation of the Kinetic Trajectory and Its Integrated Arc Coverage

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Gradientology
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ABSTRACT

Essays I through V established and locked the complete foundational, informational, geometric, transformational, and regulatory architecture of the Gradientology. Essay III derived the Registration Sphere $S^2(0.6)$ and the solid angle $\Omega = EC/F^2 = 14/9 \approx 1.556$ sr using a planar approximation per Chronon. Essay III noted explicitly that the exact integrated arc coverage of the helical worldline over multiple windings was deferred. The present essay closes Gap G-4 in full. The derivation proceeds in nine staged closures. Stage one proves the Intersection Crisis: in any manifold of dimension $d \leq 2$, a self-referential kinetic loop is forced to intersect its own path by the Jordan Curve Theorem and the non-terminating residue $\varepsilon_{snap} = 1/30$. Stage two decomposes the worldline into its three structurally independent components from the primitives: E-component (cosine, generative drive), C-component (sine, constraint), F-component (linear axial advance, registration). Stage three derives the angular frequency ω from the area sweep condition: the worldline must sweep area $A_{EC} = EC = 14/25$ per Chronon in the rotational plane, giving $\omega_{pc} = 2EC/F^2 = 2\Omega = 28/9$ rad/Chronon. Stage four derives the axial velocity $v_{pc} = \delta$ from the Speed of Causality. Stage five assembles the complete worldline $\gamma(\tau) = (F{\cdot}\cos(\omega\tau), F{\cdot}\sin(\omega\tau), c{\cdot}\tau)$ and derives all helical parameters: pitch angle $\tan(\alpha) = 3/56$, arc length per Chronon $= \sqrt{3145}/30$, helical pitch $p = 9\pi/140$, winding number per Chronon $= \Omega/\pi = 14/(9\pi)$. Stage six proves the self-avoiding property by strict axial monotonicity. Stage seven derives the exact solid angle correction: $\Omega_{exact} = (14/9)(56/\sqrt{3145})$, a correction of 0.14% from the planar approximation. Stage eight derives the exact integrated arc coverage using the spherical zone area formula $A_{zone}(N) = 18\pi N/(25\sqrt{(36+N^2)})$, giving coverage fraction $f(N) = N/(2\sqrt{(36+N^2)})$ with asymptote 1/2. After $N_{sat} = 25$ Chronons: $f(25) = 25/(2\sqrt{661}) \approx 48.6\%$. Stage nine closes Gap G-4 and establishes the geometric foundation for Essay VII. All derivations use zero free parameters.

Keywords: Helical Worldline · G-4 · Intersection Crisis · Inversive Clearance · Jordan Curve Theorem · Self-Avoiding Walk · Angular Frequency ω · Area Sweep Condition · $\omega_{pc} = 2\Omega = 28/9$ · Pitch Angle $\tan(\alpha) = 3/56$ · Arc Length $\sqrt{3145}/30$ · Loxodrome · Zone Area Formula · Coverage Fraction $f(N)$ · Northern Hemisphere Asymptote · Winding Number $= \Omega/\pi$

Derivational Profile

Table 0. Essay Derivational Profile

Attribute	Content
Essay Number	VI
Series	Gradient Cybernetics: The Calculus of Recursion
Title	The Helical Worldline on S^2 — The Exact Derivation of the Kinetic Trajectory and Its Integrated Arc Coverage
Primary Gap(s)	G-4 — Exact derivation of $\gamma(\tau)$; all helical parameters from Hardlock; exact arc coverage replacing planar approximation $\Omega=14/9$
Input Constants	$E=4/5=0.8$, $C=7/10=0.7$, $F=3/5=0.6$, $\delta=1/10=0.1$, $\tau_0=10\delta t$, $c=0.01$, $\Omega=14/9$, $\varepsilon_{snap}=1/30$, $N_{sat}=25$ [all Hardlocked, Essays I–V]
Core Derivation	$\omega_{pc}=2\Omega=28/9$ (area sweep); $\tan(\alpha)=3/56$; $L_{pc}=\sqrt{3145}/30$; $p=9\pi/140$; windings$=\Omega/\pi$; $A_{zone}(N)=18\pi N/(25\sqrt{(36+N^2)})$; $f(N)=N/(2\sqrt{(36+N^2)})$
Key Result	Exact worldline $\gamma(\tau)$ locked. $f(N_{sat}=25)=25/(2\sqrt{661})\approx 48.6\%$. Northern hemisphere asymptote 50%. G-4 CLOSED.
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1. Introduction

Essays I through V closed every foundational gap in the Gradientology. The primitive suite {E=0.8, C=0.7, F=0.6, δ=0.1} was locked in Essay I. The informational ground — I_min, δ, F as attractor, ρ=0.04, N_sat=25 — was closed in Essay II. The geometric architecture — C^3, $S^2(0.6)$, solid angle Ω=14/9 sr, resolution remainder ε_snap=1/30, d=3 uniqueness — was closed in Essay III. The transformation of F, the Fundamental Stable Knot k=3, and the Non-Equilibrium Theorem were closed in Essay IV. The Regulatory Monopoly — G=EC/F is the unique valid kinetic equation, F is the absolute Cybernetic Governor — was closed in Essay V.

One gap remained registered in the ledger: Gap G-4, the exact derivation of the kinetic worldline and its integrated arc coverage on $S^2(0.6)$. Essay III's derivation of the solid angle $\Omega = EC/F^2 = 14/9$ sr used a planar approximation, treating the EC cross-section as a flat disk perpendicular to the F-axis at each discrete Chronon step. Essay III explicitly deferred the exact computation of the helical arc over multiple windings. The present essay closes that gap in full.

Three key questions drive this essay. First: what are the exact values of all helical parameters — angular frequency ω, pitch angle α, arc length L_pc, helical pitch p, and winding number — each derived from the Hardlock with zero free parameters? Second: what is the exact integrated arc coverage on $S^2(0.6)$ after N Chronons, replacing the per-Chronon planar beam approximation with a cumulative zone formula? Third: what is the precise relationship between the beam solid angle $\Omega = 14/9$ sr (an instantaneous quantity, per Chronon) and the cumulative non-overlapping coverage (a cumulative quantity, growing with N)?

The derivation proceeds in nine stages. Sections 2 through 4 establish the Intersection Crisis ($d \leq 2$ foreclosed), decompose the worldline into its three primitive components, and derive the angular frequency from the area sweep condition. Sections 5 through 7 derive the axial velocity, assemble the complete worldline equation with all parameters, and derive pitch angle, arc length, and pitch. Section 8 proves the self-avoiding property by strict axial monotonicity. Section 9 derives the exact solid angle correction. Section 10 derives the zone area formula and exact coverage fraction, including the behaviour at N_sat = 25 and the northern hemisphere asymptote. Section 11 closes Gap G-4 and anticipates Essay VII. A note on notation: the superscript in S^2 denotes the two-dimensional spherical surface; an asterisk superscript (G^*) denotes a fixed-point value, as established in Essay IV, Section 3.1.

2. The Intersection Crisis — Formal Proof That d ≤ 2 Is Foreclosed

The Intersection Crisis is the structural proof that no self-referential kinetic loop can close without self-intersection in any manifold of dimension $d \leq 2$. It establishes d=3 as the minimum dimensionality for a self-avoiding worldline, supplementing the counting argument of Essay III (Section 9) with a direct topological proof using the Jordan Curve Theorem and the arithmetic of $\varepsilon_{snap} = 1/30$.

2.1 The Self-Intersection Requirement in d=2

The kinetic worldline must satisfy two conditions simultaneously. (i) It must close: the worldline must return to configurations consistent with its own prior output in order to achieve the Recursive Closure condition of Essay IV. (ii) It must avoid self-intersection: distinct Chronons must occupy distinct lattice coordinates; otherwise two different registration events would be indistinguishable (violating Essay II, D-5.1: the discriminability requirement, $r_{min} = 1/\sqrt{3}$).

In d=2 these two conditions are formally incompatible. The Jordan Curve Theorem states that any simple closed curve in $\mathbb{R}^2$ divides the plane into exactly two regions: interior and exterior. For a kinetic worldline that must close (satisfy Recursive Closure), the curve must form a simple closed loop. Once the loop is closed, any subsequent advance must pass through interior or exterior, crossing the loop itself. The worldline cannot continue without intersecting its prior path.

THEOREM — T-2.1 — The Intersection Crisis in d=2

In any two-dimensional manifold, a self-referential kinetic loop satisfying the Recursive Closure condition of Essay IV is topologically forced to intersect its own path within finitely many Chronons.

Proof: By the Jordan Curve Theorem, any simple closed curve C in $\mathbb{R}^2$ divides the plane into interior I(C) and exterior E(C). For Recursive Closure, the worldline must complete at least k windings before re-using any coordinate. In d=2, the winding brings the worldline back to the vicinity of its starting point, and the next winding must either enter I(C) or remain in E(C). In either case it must cross C itself — a self-intersection. The resolution remainder $\varepsilon_{snap} = 1/30$ accumulates as an irresolvable phase offset in d=2. After 30 windings the accumulated offset $30 \times \varepsilon_{snap} = 1 = 10\delta$ requires 10 lattice steps of re-alignment, all of which cause self-intersection in the 2D plane. FORECLOSED. □

2.2 The Resolution Remainder Arithmetic in d=2

The Intersection Crisis in d=2 is made arithmetically precise by the resolution remainder. From Essay III (D-9.1): $\varepsilon_{snap} = EC/F - 9/10 = 14/15 - 9/10 = 28/30 - 27/30 = 1/30$. This residual arises because the kinetic ratio $EC/F = 14/15$ is non-terminating on the $\delta = 0.1$ lattice.

In d=2, after each complete winding the phase offset 2π is replaced by $2\pi + \varepsilon_{snap}$ on the δ-lattice. The accumulated phase offset after k windings is $k/30$. For this to equal zero modulo $\delta = 0.1$, k must be a multiple of 3 (from $3\varepsilon_{snap} = \delta$). But in d=2, the worldline must cross its own path during the process of accumulating these k windings. The self-intersection occurs where winding k returns to the vicinity of the starting point and the phase offset $k/30 \leq \varepsilon_{snap}$ forces a lattice collision.

DERIVATION — D-2.1 — The d=1 Foreclosure

In d=1 the worldline is a straight line or a closed interval. A closed interval in d=1 has only two boundary points; a straight line has no boundary. Neither geometry supports a self-referential loop: a loop in d=1 requires the worldline to reverse direction, producing a self-intersection at the turning point (the worldline occupies the same coordinate twice).

Additionally: in d=1 the generative dyad EC cannot be represented. Two primitives orthogonal in C^3 cannot both appear in a one-dimensional space without collapsing. The area $A_{EC} = EC = 0.56$ swept per Chronon is identically zero in d=1. With no area swept in the EC-plane, the kinetic engine $G = EC/F$ produces output that does not depend on C (C collapses onto the E-axis). The Triadic Mandate is violated. FORECLOSED. □

2.3 The d=3 Resolution

The third dimension — the F-axis — provides the topological relief that d=2 cannot supply. In d=3 the worldline advances along the F-axis simultaneously with its rotational sweep in the EC plane. This axial advance means consecutive windings occur at different axial positions: winding n is at axial coordinate $n\delta$, winding n+1 at $(n+1)\delta$. The two windings are separated by exactly one lattice step δ in the F-axis direction, so they never occupy the same 3D lattice coordinate.

The clearance condition: in d=3, the worldline returning to the vicinity of its starting azimuthal position has already advanced by one lattice step δ along the F-axis per Chronon. After k=3 Chronons (the Fundamental Stable Knot from Essay IV), the accumulated phase offset $3\varepsilon_{snap} = \delta$ is cleared by the axial advance: the worldline has moved 3δ along the F-axis, and the $3\varepsilon_{snap} = \delta$ phase offset snaps to the next lattice point without collision. The third dimension absorbs the residual that d=2 cannot discharge.

THEOREM — T-2.2 — d=3 Is the Unique Dimensionality Resolving the Intersection Crisis

d=1: FORECLOSED. No two-dimensional area, Triadic Mandate violated, self-intersection at every reversal.

d=2: FORECLOSED. Jordan Curve Theorem forces self-intersection; ε_{snap} accumulates without discharge.

d=3: UNIQUE RESOLUTION. The F-axis provides the over-under clearance. Consecutive windings are separated by one lattice step δ in the axial direction. The accumulated offset $3\varepsilon_{snap} = \delta$ is cleared by the 3-Chronon axial advance 3δ. The worldline is self-avoiding.

d>3: FORECLOSED (Essay III, D-2.2; Essay I): additional dimensions provide unconstrained axes not grounded in any primitive, creating thermodynamic sinks that dissipate the Kinetostatic Margin $\Phi = +0.002$ before any registration event occurs.

d=3 is the unique dimensionality satisfying both the Intersection Crisis resolution (Recursive Closure without self-intersection) and the Triadic Mandate (all three primitives independently represented). □

3. Decomposition of the Worldline — Three Independent Components from Primitives

The worldline $\gamma(\tau)$ in C^3 is a curve parametrised by continuous time τ. Its structure must be derived from the three Hardlocked primitives, one primitive per configuration-space axis (Essay III: the three orthogonal basis vectors $\hat{e}$, $\hat{c}$, $\hat{f}$ correspond to E, C, F respectively). The worldline has three structurally independent components, each determined by a different primitive.

3.1 The E-Component — Generative Drive as Cosine

Systematisation (E = 0.8) is the generative drive: the source of potential whose role is to initiate. In the kinetic worldline, E contributes the cosine (real) component of the rotational motion in the EC plane. The cosine represents the drive's phase-leading character: at $\tau=0$, the worldline starts at maximum drive-projection ($x = F$, the registration amplitude). The E-component is $x(\tau) = F \cdot \cos(\omega\tau)$.

Why cosine rather than sine? At $\tau=0$, the system initialises from the Phase I ground state in which the generative drive E was the first to be established (logical chronology: E precedes C precedes F, from Essay I, T-2.1). The cosine is the phase-zero component: it equals its maximum F at $\tau=0$. The sine would require a phase offset, implying a prior history that contradicts the Primordial Axiom of Relationality (no pre-existing entity). The cosine start is derivationally required.

3.2 The C-Component — Constraint as Sine

Constraint (C = 0.7) is the limitative boundary: the structural threshold that negates unbounded potential into specific morphospace. In the kinetic worldline, C contributes the sine (imaginary) component of rotational motion. The sine represents the 90° phase lag of the constraint behind the drive: the constraint responds to the drive, imposing a boundary on what the drive has initiated. At $\tau=0$: $y(0) = F \cdot \sin(0) = 0$. The C-component is $y(\tau) = F \cdot \sin(\omega\tau)$.

The 90° phase separation between E (cosine) and C (sine) is the geometric expression of their orthogonality: from Essay III (D-2.1), $\hat{e} \cdot \hat{c} = 0$. Two orthogonal primitives with a 90° phase difference in their contributions to circular motion produce a circle. The radius of this circle is not E or C individually, but F = 0.6: the Registration primitive defines the discriminability boundary. Any point on the worldline at radius less than F would be below the noise floor (Essay II, D-5.1: $r_{min} = 1/\sqrt{3} \approx 0.577$). The worldline must stay at distance F from the origin in the rotational plane.

3.3 The F-Component — Registration as Axial Advance

Registration ($F = 0.6$) occupies the denominator of the kinetic equation $G = EC/F$ and defines the propagation axis. In Phase II, F is the axis of temporal advance: the worldline propagates along the F-axis at the Speed of Causality $c = \delta/\tau_0 = 0.01$. The z-component of the worldline is $z(\tau) = c{\cdot}\tau$, a strictly monotone linear function of time.

Why linear and not accelerating? In Phase II (constant flux, Essay I, Section 5), the axial advance is uniform at the Speed of Causality. The Speed of Causality is the grid update rate: one lattice step δ per Chronon τ_0 (Essay I, Equation Set 7: $c = \delta/\tau_0 = 0.1/10 = 0.01$). The Phase II worldline advances uniformly. The $d^2G/dt^2 > 0$ Non-Equilibrium Theorem (Essay IV, T-6.1) is a Level n=3 mandate on the RATE of kinetic output, not on the axial position of the Phase II worldline.

PRINCIPLE — P-3.1 — Structural Derivation of the Three Worldline Components

The three components of the kinetic worldline are each derived from a different primitive:

$x(\tau) = F{\cdot}\cos(\omega\tau)$: E-component. Amplitude F (registration boundary); cosine from E's phase-leading generative role and $\tau=0$ initialisation; frequency ω from area sweep (Section 4).

$y(\tau) = F{\cdot}\sin(\omega\tau)$: C-component. Amplitude F; sine from C's 90°-lagging limitative role; same ω.

$z(\tau) = c{\cdot}\tau$: F-component. Speed of Causality c as axial velocity; strictly linear Phase II constant flux.

The radius $F=0.6$ is shared by all rotational components: the Registration primitive defines both the discriminability boundary (why the amplitude is F) and the propagation axis (why z advances along F). □

4. The Angular Frequency ω — Derivation from the Area Sweep Condition

The angular frequency ω is the rate at which the worldline rotates in the EC plane. Its value determines the winding number per Chronon, the helical pitch, the arc length, and the exact surface coverage. It must be derived from the Hardlock with zero free parameters.

4.1 The Area Sweep Condition

The worldline sweeps area in the EC plane as it rotates. Per Chronon, this area must equal the EC cross-sectional area $A_{EC} = E \times C = 14/25 = 0.56$ (Essay III, D-8.1). This is the structural condition connecting the worldline's rotational dynamics to the generative dyad: the worldline must sweep exactly the area that the kinetic engine generates per Chronon.

The circular sector swept in the EC plane during one Chronon (angular advance $\theta = \omega_{pc}$, where subscript pc denotes per-Chronon) has area $(1/2)r^2\theta = (1/2)F^2\omega_{pc}$. Setting this equal to A_{EC}:

EQUATION SET 1 — AREA SWEEP CONDITION

$$A_{sector} = (1/2) \times r^2 \times \omega_{pc} = (1/2) \times F^2 \times \omega_{pc} \quad (1)$$

└ *sector area per Chronon with r=F, angular advance ω_{pc}*

$$\text{Setting } A_{sector} = A_{EC}: \ (1/2) \times F^2 \times \omega_{pc} = E \times C = 14/25 \quad (2)$$

└ *area sweep condition: worldline sweeps EC area per Chronon*

$$\omega_{pc} = 2(E \times C)/F^2 = 2 \times (14/25)/(9/25) = 2 \times 14/9 = 28/9 \text{ radians per Chronon} \quad (3)$$

└ *angular advance per Chronon — LOCKED*

$$\omega_{pc} = 2\Omega = 2 \times (EC/F^2) = 28/9 \quad (4)$$

└ *ω_{pc} equals twice the solid angle $\Omega = 14/9$: structural identity*

The identity $\omega_{pc} = 2\Omega$ is structurally significant and exact. The solid angle $\Omega = EC/F^2$ is the area-to-radius-squared ratio. The sector area formula introduces the factor of 1/2: $A_{sector} = (1/2)F^2\omega_{pc} = (1/2)F^2 \times 2\Omega = F^2\Omega = F^2 \times (EC/F^2) = EC$. So the area sweep condition is automatically satisfied when $\omega_{pc} = 2\Omega$. The angular frequency per Chronon is exactly twice the solid angle per Chronon. This algebraic identity is a consequence of the sector area formula and holds for any sector, not just for the Veldt's specific values.

4.2 The Winding Number Per Chronon

The winding number per Chronon is the number of complete revolutions in one Chronon:

EQUATION SET 2 — WINDING NUMBER PER CHRONON

$$\text{Windings per Chronon} = \omega_{pc} / (2\pi) = (28/9) / (2\pi) = 14/(9\pi) \quad (1)$$

└─ *revolutions per Chronon — LOCKED*

$$14/(9\pi) = \Omega/\pi \quad (2)$$

└─ *winding number per Chronon = Ω/π: structural identity*

$$14/(9\pi) \approx 0.4952 \text{ revolutions per Chronon} \quad (3)$$

└─ *the worldline completes slightly less than half a revolution per Chronon*

$$\text{Chronons per full revolution} = (2\pi)/\omega_{pc} = 9\pi/14 \approx 2.02 \text{ Chronons per revolution} \quad (4)$$

└─ *one full revolution takes approximately 2 Chronons*

The winding number per Chronon is $\Omega/\pi = 14/(9\pi) \approx 0.495$ — slightly less than half a revolution per Chronon. This non-integer value is structurally important: if one full revolution took exactly 2 Chronons (or any integer number), the worldline would be periodic and would repeat the same lattice positions with a phase offset per period. The value $9\pi/14$ is irrational, ensuring the worldline is quasi-periodic: it never exactly repeats any lattice angular coordinate.

THEOREM — T-4.1 — The Angular Frequency Is Unique

$\omega_{pc} = 28/9$ radians per Chronon is the unique angular frequency satisfying the area sweep condition $A_{sector} = A_{EC}$ given:

(i) The worldline radius must equal $F = 0.6$ (registration boundary, Essay III, D-4.1).

(ii) The swept area per Chronon must equal $A_{EC} = EC = 14/25$.

(iii) $\omega_{pc} = 2A_{EC}/F^2 = 2EC/F^2 = 2\Omega$ follows algebraically; no free parameter is introduced.

No alternative value of ω_{pc} satisfies (ii) with $r=F$ and $A_{EC}=EC$. □

5. The Axial Component — The Speed of Causality and the F-Axis Advance

The axial component $z(\tau) = c \cdot \tau$ requires deriving the axial velocity $v = c$ and expressing the per-Chronon advance. Both values were locked in Essay I.

5.1 The Speed of Causality

The Speed of Causality $c = \delta/\tau_0$ is the grid update rate: the system advances by one lattice step per Chronon. From Essay I: the Chronon $\tau_0 = 10\delta t$ is the minimum duration of one complete kinetic computation ($N_steps = 10$ atomic lattice operations). Therefore $c = \delta/\tau_0 = 0.1/10 = 0.01$ in natural units. The per-Chronon axial advance is:

EQUATION SET 3 — AXIAL ADVANCE PER CHRONON

$$v_pc = c \times \tau_0 = 0.01 \times 10\delta t = 0.1\delta t = \delta \tag{1}$$

└ *axial advance per Chronon = one lattice step δ — LOCKED*

$$\delta = 1/10 = 0.1 \tag{2}$$

└ *lattice grain — LOCKED*

$$F / v_pc = F / \delta = (3/5) / (1/10) = 6 \tag{3}$$

└ *sphere radius spans exactly 6 axial lattice steps: F = 6δ*

$$\text{After 6 Chronons: } z = 6\delta = 0.6 = F \tag{4}$$

└ *the worldline reaches the full sphere radius axially after 6 Chronons*

The ratio $F/v_pc = F/\delta = 6$ is structurally significant. It determines the axial scale of the sphere: the sphere radius $F = 0.6$ spans exactly 6 lattice steps. After exactly 6 Chronons, the axial coordinate $z = 6\delta = F$. This is the moment when the loxodrome's leading edge has advanced by the full sphere radius along the F-axis, corresponding to 45° colatitude on the sphere.

5.2 The Axial Advance and the d=3 Clearance

The axial advance per Chronon being exactly δ is not a coincidence. Essay III proved that in d=3, three consecutive Chronon steps clear the resolution remainder: $3\varepsilon_{snap} = 3/30 = 1/10 = \delta$. Each Chronon contributes exactly 1/3 of a lattice step to the remainder clearance, and advances exactly $\delta = 1/10$ along the F-axis. After 3 Chronons, the worldline has advanced 3δ = the exact amount needed to clear the resolution remainder. The axial advance and the remainder clearance are geometrically coupled: the worldline advances along the F-axis at the rate that enables the d=3 clearance mechanism.

COROLLARY — C-5.1 — The Axial Advance Enables d=3 Clearance

$v_{pc} = \delta$ is the unique axial velocity consistent with the d=3 clearance mechanism:

3 Chronons × $v_{pc} = 3\delta$ = axial advance for one resolution remainder clearance.

3 Chronons × $\varepsilon_{snap} = 3/30 = 1/10 = \delta$ = one lattice step closed.

The per-Chronon axial advance equals the lattice grain by structural necessity. If $v_{pc} \neq \delta$, the Fundamental Stable Knot k=3 clearance cycle would fail: the worldline would advance a non-integer number of lattice steps per clearance period, producing irresolvable remainder accumulation. $v_{pc} = \delta$ is uniquely consistent with k=3. □

6. The Complete Worldline Equation — Assembly and Foreclosure of Alternatives

With all three components derived, the complete worldline equation can be assembled. This section forecloses the two principal alternative trajectory shapes: the closed circular orbit and the straight-line path, confirming that the helix is the unique trajectory consistent with all structural requirements.

6.1 The Worldline Equation

EQUATION SET 4 — THE COMPLETE WORLDLINE EQUATION

$$\gamma(\tau) = (F\cdot\cos(\omega\tau),\ F\cdot\sin(\omega\tau),\ c\cdot\tau) \quad (1)$$

└ *the kinetic helicon in C^3 — LOCKED*

$$r = F = 3/5 = 0.6 = 6\delta \quad (2)$$

└ *helix radius = Registration primitive — LOCKED*

$$\omega_{pc} = 2\Omega = 2(EC/F^2) = 28/9 \text{ radians per Chronon} \quad (3)$$

└ *angular frequency from area sweep condition — LOCKED*

$$v_{pc} = c \times \tau_0 = \delta = 1/10 \text{ per Chronon} \quad (4)$$

└ *axial velocity = Speed of Causality × Chronon duration — LOCKED*

$$|\gamma(\tau)| = \sqrt{(F^2 + c^2\tau^2)} \quad (5)$$

└ *distance from origin: increases with τ as worldline spirals from equatorial plane*

The worldline $\gamma(\tau)$ is a circular helix of radius $F = 0.6$ in the EC plane, advancing along the F-axis at velocity $v = c = 0.01$. All four parameters (F, ω_{pc}, c, v_{pc}) are Hardlocked: F from Essay II as the information-theoretic attractor; $\omega_{pc} = 28/9$ from the area sweep condition (Section 4); c from Essay I as the grid update rate; $v_{pc} = \delta$ from the Speed of Causality and Chronon duration. Zero free parameters.

6.2 Why Circular, Not Elliptical

Essay III noted that $E \neq C$ ($E - C = \delta = 0.1$) makes the cross-sectional sweep elliptical in the sense that the helicon's beam shape has an asymmetric aspect. Why is the worldline radius circular ($r = F$ for both x and y components), not $r_x = E = 0.8$ and $r_y = C = 0.7$?

The worldline radius must equal the Registration primitive $F = 0.6$ because F defines the discriminability boundary: the minimum distance from the origin at which a signal is distinguishable from noise (Essay II, D-5.1). Any point on the worldline must be at least at distance F from the origin to be registerable. If the radius in the E-direction were $E = 0.8$ and in the C-direction were $C = 0.7$, the worldline would trace an ellipse. An ellipse in C^3 has a minimum radius equal to its semi-minor axis $C = 0.7 > F = 0.6$, so both semi-axes exceed the discriminability floor. But the SHAPE of the cross-sectional swept area would be elliptical (area $= \pi \times E \times C = \pi \times 0.56$), not a circle of area $\pi F^2 = \pi \times 0.36$. The area of the ellipse $E \times C = 0.56$ is greater than $\pi F^2 = 0.36$: the helicon's beam would overlap itself radially.

The resolution: the worldline radius is F (the discriminability threshold), not E or C. The AREA swept per Chronon is $EC = 0.56$ (the generative dyad product). These are two distinct geometric quantities. The worldline traces a circle of radius F while sweeping area EC per Chronon. The asymmetry $E \neq C$ appears in the RATE at which the worldline sweeps different sectors (the drive-dominated and constraint-dominated phases), not in the radius of the trajectory.

DERIVATION — D-6.1 — Foreclosure of the Closed Circular Orbit ($v_{pc} = 0$)

A closed circular orbit in the EC plane (no axial advance: $v_{pc} = 0$) sweeps A_{EC} per Chronon but closes after each revolution. The resolution remainder $\varepsilon_{snap} = 1/30$ has accumulated but cannot be discharged: in d=2 (the EC plane), there is no third dimension to absorb it. By T-2.1, the next revolution produces a self-intersection. FORECLOSED.

Additionally: the closed orbit has $z(\tau) = 0$ for all τ. The worldline never advances along the F-axis. But $v_{pc} = c\tau_0 = \delta > 0$ by the Speed of Causality (Essay I). A zero axial velocity violates the constraint $v_{pc} > 0$. The circular orbit is doubly foreclosed: by the Intersection Crisis and by the Speed of Causality. FORECLOSED. □

DERIVATION — D-6.2 — Foreclosure of the Straight-Line Path ($\omega_{pc} = 0$)

A straight-line path along the F-axis ($\omega_{pc} = 0$, no rotation) has $x(\tau) = F$ (constant), $y(\tau) = 0$, $z(\tau) = c\tau$. This worldline sweeps zero area in the EC plane.

Swept area per Chronon $= 0 \neq A_{EC} = 0.56$. The area sweep condition is violated: the kinetic engine contributes nothing to the EC cross-section. The Constraint primitive C does not appear in the worldline ($y(\tau) = 0$ always), violating the Triadic Mandate. A straight-line path along the F-axis omits the C-component entirely. FORECLOSED. □

7. Helical Parameters — Pitch Angle, Arc Length, Pitch, and Winding Structure

With the worldline equation fully derived, all helical parameters follow as zero-free-parameter consequences of the Hardlock. These parameters characterise the geometry of the helix in C^3.

7.1 The Pitch Angle

The pitch angle α is the angle between the worldline tangent and the equatorial plane (the EC plane). It measures the steepness of the helix: a small pitch angle means a nearly-flat helix with many windings per unit axial advance.

EQUATION SET 5 — THE PITCH ANGLE

$$\tan(\alpha) = v_{pc} / (F \times \omega_{pc}) \quad (1)$$
└ *pitch angle: ratio of axial advance to circumferential arc per Chronon*

$$v_{pc} = \delta = 1/10 \quad (2)$$
└ *axial advance per Chronon — LOCKED*

$$F \times \omega_{pc} = (3/5) \times (28/9) = 84/45 = 28/15 \quad (3)$$
└ *circumferential arc per Chronon in EC plane*

$$\tan(\alpha) = (1/10) / (28/15) = (1/10) \times (15/28) = 15/280 = 3/56 \quad (4)$$
└ *exact rational pitch angle tangent — LOCKED*

$$\alpha = \arctan(3/56) \approx 3.07^\circ \quad (5)$$
└ *pitch angle: shallow helix, nearly equatorial*

The exact rational value $\tan(\alpha) = 3/56$ has structural content. The numerator 3 is the Fundamental Stable Knot $k = 3$ (Essay IV, T-5.1). The denominator $56 = F\omega_{pc} \times 30$, where $30 = k \times \tau_0/\delta t = 3 \times 10$ connects the Fundamental Stable Knot, the Chronon duration, and the lattice grain. In the frame scaled by 30δ per Chronon, the pitch angle is simply $\tan(\alpha) = k/56$: the numerator is precisely the Fundamental Stable Knot.

This is not decorative: the Fundamental Stable Knot k=3 appears in the pitch angle because k was itself derived from the condition $3\varepsilon_{snap} = \delta$, which is the same condition that makes $v_{pc} = \delta$ the axial advance.

7.2 The Arc Length Per Chronon

The arc length of the worldline per Chronon is the Euclidean length of the helical segment traced in one Chronon:

EQUATION SET 6 — THE ARC LENGTH PER CHRONON

$$L_{pc} = \sqrt{((F\omega)_{pc})^2 + v_{pc}^2} \quad (1)$$

└ *Pythagorean arc length combining circumferential and axial advances*

$$(F\omega)_{pc}^2 = (28/15)^2 = 784/225 \quad (2)$$

└ *circumferential component squared*

$$v_{pc}^2 = (1/10)^2 = 1/100 = 9/900 \quad (3)$$

└ *axial component squared; LCD: 784/225 = 3136/900*

$$L_{pc} = \sqrt{((3136+9)/900)} = \sqrt{(3145/900)} = \sqrt{3145} / 30 \quad (4)$$

└ *exact arc length per Chronon — LOCKED*

$$56^2 = 3136,\ 3^2 = k^2 = 9,\ 3136+9 = 3145 \quad (5)$$

└ *so $L_{pc} = \sqrt{(56^2 + k^2)} / 30$: hypotenuse of (56, k=3) scaled by 30*

$L_{pc} = \sqrt{3145}/30 \approx 56.081/30 \approx 1.869$ lattice units per Chronon. The worldline advances approximately 1.867 units circumferentially and 0.1 units axially per Chronon. The dominant motion is rotational. The structural form $L_{pc} = \sqrt{(56^2 + k^2)}/30$ again exhibits the Fundamental Stable Knot k=3 as the axial numerator in the Pythagorean decomposition.

7.3 The Helical Pitch

The helical pitch p is the axial advance per complete revolution, the distance along the F-axis between adjacent windings:

EQUATION SET 7 — THE HELICAL PITCH

$$p = v_{pc} \times (\text{Chronons per revolution}) = \delta \times (2\pi/\omega_{pc}) \quad (1)$$
└ *pitch = axial advance per full revolution*

$$2\pi/\omega_{pc} = 2\pi / (28/9) = 9\pi/14 \quad (2)$$
└ *Chronons per full revolution — LOCKED*

$$p = \delta \times 9\pi/14 = (1/10) \times 9\pi/14 = 9\pi/140 \quad (3)$$
└ *helical pitch in lattice units — LOCKED*

$$p = 9\pi/140 \approx 0.202 \text{ lattice units per revolution} \quad (4)$$
└ *approximately 2δ inter-winding separation*

The helical pitch $p = 9\pi/140 \approx 0.202$ lattice units means adjacent windings are separated by about $2\delta = 0.2$ lattice units. The helix is tightly wound: approximately 5 complete revolutions fit within one lattice unit of axial advance. This tight winding explains why the cumulative coverage grows slowly despite the large beam solid angle: each new winding is very close to the previous one, covering almost entirely overlapping territory on the sphere.

7.4 The Winding Structure and the k=3 Connection

EQUATION SET 8 — WINDING STRUCTURE AND k=3 CONNECTION

$$\text{Revolutions in k=3 Chronons} = 3 \times 14/(9\pi) = 14/(3\pi) \approx 1.485 \text{ revolutions} \quad (1)$$
└ *k=3 Chronons produce 1.485 revolutions: between 1 and 2 full windings*

$$\text{Axial advance in k=3 Chronons} = 3v_{pc} = 3\delta = 0.3 \text{ lattice units} \quad (2)$$
└ *exactly 3 lattice steps: clears ε_snap*

$$3\varepsilon_{snap} = 3/30 = 1/10 = \delta\text{: remainder discharged in k=3 Chronons} \quad (3)$$
└ *arithmetic closure of stable knot*

$$14/(3\pi) \text{ is irrational: helix is quasi-periodic, not periodic} \quad (4)$$
└ *non-integer winding: no exact return to starting position*

The non-integer winding count $14/(3\pi) \approx 1.485$ after k=3 Chronons confirms that the stable knot k=3 is an ARITHMETIC closure (the resolution remainder ε_{snap} is discharged) and not a GEOMETRIC closure (the worldline does not return to the same angular position). If the helix were geometrically closed after k=3 Chronons, consecutive windings would overlap exactly, producing periodic self-intersections in the rotational plane. The quasi-periodicity is structurally required by the self-avoiding property.

8. The Self-Avoiding Walk — Formal Proof of Non-Intersection

The self-avoiding property is the structural guarantee that the worldline never re-occupies any lattice coordinate it has previously occupied. This is the geometric expression of the Arrow of Time: if the worldline revisited a prior coordinate, the system would lose the ability to distinguish past from present states, destroying Ontic Irreversibility (Essay IV, T-7.1).

8.1 Continuous Non-Intersection

THEOREM — T-8.1 — The Worldline Is Strictly Self-Avoiding

The worldline $\gamma(\tau) = (F\cdot\cos(\omega\tau), F\cdot\sin(\omega\tau), c\cdot\tau)$ is injective: for all $\tau_1 \neq \tau_2$, $\gamma(\tau_1) \neq \gamma(\tau_2)$.

Proof: Suppose $\gamma(\tau_1) = \gamma(\tau_2)$. Then equality of the z-components gives $c\cdot\tau_1 = c\cdot\tau_2$. Since $c = 0.01 \neq 0$, this implies $\tau_1 = \tau_2$. Therefore no two distinct parameter values give the same worldline point. The worldline is injective: it never revisits any point in C^3. The proof uses only $c \neq 0$ (the Speed of Causality is strictly positive), which is Hardlocked ($c = \delta/\tau_0 = 0.01 > 0$). □

The proof is structurally simple and relies only on the axial component. The rotational components (x and y) could in principle coincide at $\tau_2 = \tau_1 + 2\pi/\omega$ (after exactly one full revolution), but the axial component at τ_2 differs: $c\cdot\tau_2 = c(\tau_1 + 2\pi/\omega) = c\tau_1 + 2\pi c/\omega \neq c\tau_1$ since $c, \omega > 0$. The worldline spirals strictly upward: each pass through any azimuthal angle occurs at a strictly higher axial coordinate than all previous passes.

8.2 Lattice Non-Intersection

On the discrete δ-lattice, the self-avoiding property must be stated in terms of lattice coordinates. Two worldline points $\gamma(n_1\tau_0)$ and $\gamma(n_2\tau_0)$ occupy the same lattice site if and only if their rounded z-coordinates AND their rounded EC-plane coordinates both agree. The z-coordinate argument from T-8.1 establishes that z-lattice coordinates are distinct as long as $n_1 \neq n_2$, since the axial advance per Chronon is exactly $v_{pc} = \delta$ = one lattice step. Distinct Chronons always occupy distinct z-lattice coordinates.

COROLLARY — C-8.1 — Lattice Self-Avoidance of the Worldline

On the δ-lattice, distinct Chronon indices $n_1 \neq n_2$ always produce distinct lattice positions:

$z(n_1\tau_0) = n_1\delta \neq n_2\delta = z(n_2\tau_0)$ since $n_1 \neq n_2$ and $\delta > 0$.

The z-lattice coordinate uniquely identifies each Chronon. The worldline never revisits any lattice site at the δ-resolution. This is the geometric expression of Ontic Irreversibility: the lattice history of the worldline is strictly forward-ordered with no repetitions. □

8.3 The Arrow of Time as Geometric Consequence

The Arrow of Time was derived in Essay I (Section 5) as the non-injective (many-to-one) character of the Registration Snap: the mapping $G_{raw} = 14/15 \to G_{snap} = 0.9$ destroys $\Delta H = 0.02$ bits per Chronon. In the context of the worldline, the Arrow of Time has a geometric counterpart: the worldline is geometrically oriented, advancing in the positive F-axis direction with no backward movement. The axial coordinate $z(n) = n\delta$ is a strictly increasing sequence of distinct lattice values, providing a unique temporal index for every lattice site in C^3 ever occupied by the worldline.

The geometric Arrow of Time (strict z-monotonicity) and the informational Arrow of Time ($\Delta H = 0.02$ information loss per snap) are independent derivations of the same structural fact: the Veldt cannot run backward. The z-coordinate stamps every registration event with a unique temporal index; the ΔH stamps every snap with an irrecoverable information cost. Both arrows are consequences of the same primitive F: F defines the propagation axis (z-component) and the denominator of $G = EC/F$ (producing the snap $G_{raw} \to G_{snap}$ and the associated ΔH). F is the source of both arrows.

9. The Exact Solid Angle — Correcting the Planar Approximation

Essay III's solid angle $\Omega = EC/F^2 = 14/9 \approx 1.556$ sr was derived under the planar approximation: the EC cross-sectional area was treated as a flat disk perpendicular to the F-axis. The exact solid angle accounts for the pitch of the helix: the worldline's tangent is tilted by the pitch angle α from the perpendicular, so the beam element is a tilted strip, not a flat disk.

9.1 The Planar Approximation and Its Scope

The planar approximation treated $A_{EC} = 0.56$ as a flat element normal to the F-axis at distance $r = F = 0.6$. The solid angle formula for a flat element A at distance r (element normal to radial direction): $\Omega = A/r^2$. Applied to A_{EC} and $r = F$: $\Omega_{planar} = EC/F^2 = 14/9$. This is the solid angle subtended by the beam (the instantaneous cross-sectional 'spot' of the helicon) as seen from the origin. The approximation is exact to leading order in α since $\alpha = \arctan(3/56) \approx 3.07°$ is small.

9.2 The Exact Solid Angle Per Chronon

The helical arc segment per Chronon is a tilted strip. The solid angle subtended by a tilted area element A at distance r, tilted by angle β from the normal direction, is $\Omega = A \cdot \cos(\beta)/r^2$. The tilt angle equals the pitch angle α:

EQUATION SET 9 — THE EXACT SOLID ANGLE PER CHRONON

$$\Omega_{exact} = \Omega_{planar} \times \cos(\alpha) \tag{1}$$
└ exact solid angle = planar approximation × cosine of pitch angle

$$\cos(\alpha) = 1/\sqrt{(1 + \tan^2(\alpha))} = 1/\sqrt{(1 + 9/3136)} = 56/\sqrt{3145} \tag{2}$$
└ exact cosine: tan(α) = 3/56 so 1 + (3/56)² = 3145/3136

$$\Omega_{exact} = (14/9) \times (56/\sqrt{3145}) \tag{3}$$
└ exact solid angle per Chronon — LOCKED

$$\Omega_{exact} \approx (14/9) \times (56/56.081) \approx 1.5534 \text{ sr} \tag{4}$$
└ numerical: 1.5534 sr vs Ω_planar = 1.5556 sr

$$\Delta\Omega/\Omega_\text{planar} = 1 - 56/\sqrt{3145} \approx 0.00144 = 0.144\% \tag{5}$$

└— *relative correction from planar to exact: 0.144%*

The correction is tiny: approximately 0.144% of Ω_planar. Essay III's use of the planar approximation was structurally justified: the pitch angle $\alpha = \arctan(3/56) \approx 3.07°$ is small because the axial advance per Chronon ($\delta = 0.1$) is much smaller than the circumferential arc ($28/15 \approx 1.867$). The helix is shallow, the pitch is small, and the beam element is nearly perpendicular to the radial direction.

COROLLARY — C-9.1 — The Planar Approximation Is Accurate to 0.15%

The exact solid angle $\Omega_\text{exact} = (14/9)(56/\sqrt{3145})$ differs from the planar value by a correction:

$1 - \cos(\alpha) = 1 - 56/\sqrt{3145} \approx 0.00144$.

Relative correction $\Delta\Omega/\Omega_\text{planar} \approx 0.144\%$, well below any physically significant threshold.

Essay III's planar approximation $\Omega = 14/9$ is accurate to better than 0.2% of the exact value. The correction is formally derived and now locked, but the approximation was correctly characterised in Essay III as a planar-projection approximation that would be refined by the exact helical geometry. □

10. The Exact Integrated Arc Coverage — Zone Area Formula and Coverage Fraction

Having derived all helical parameters, the exact integrated arc coverage of the worldline's projection on $S^2(0.6)$ can now be computed. This replaces the per-Chronon planar beam approximation with a cumulative zone area formula.

10.1 The Loxodrome on $S^2(0.6)$

The worldline $\gamma(\tau)$ does not lie on $S^2(0.6)$: its distance from the origin is $|\gamma(\tau)| = \sqrt{(F^2 + c^2\tau^2)}$, which exceeds F for all $\tau > 0$ as the worldline spirals away from the equatorial plane. The radial projection of the worldline onto $S^2(0.6)$ is:

EQUATION SET 10 — THE LOXODROME (RADIAL PROJECTION ONTO S^2)

$$\tilde{\gamma}(\tau) = F \times \gamma(\tau) / |\gamma(\tau)| \quad (1)$$
└ *radial projection: normalise worldline to sphere radius F*

$$|\gamma(\tau)| = \sqrt{(F^2 + c^2\tau^2)} \quad (2)$$
└ *worldline distance from origin*

$$\text{z-component of } \tilde{\gamma}\text{: } F{\cdot}c\tau/\sqrt{(F^2+c^2\tau^2)} \quad (3)$$
└ *axial projection: increases toward F as τ increases*

$$|\tilde{\gamma}(\tau)| = F \text{ for all } \tau \quad (4)$$
└ *the projection always lies on $S^2(0.6)$ by construction*

The projected curve $\tilde{\gamma}(\tau)$ traces a loxodrome on $S^2(0.6)$ — a curve that spirals from the equator toward the north pole, making a constant angle with every meridian (a constant bearing). In spherical coordinates (colatitude θ from north pole, azimuth = $\omega\tau$), the loxodrome starts at the equator ($\theta = \pi/2$ at $\tau = 0$) and spirals toward the north pole ($\theta \to 0$ as $\tau \to \infty$), making progressively tighter spirals as latitude increases.

10.2 The Spherical Zone Area Formula

After N Chronons, the worldline has advanced by $Nv_{pc} = N\delta$ along the z-axis. The colatitude θ_N of the loxodrome's leading edge satisfies $\cos(\theta_N) = N\delta/\sqrt{(F^2 + N^2\delta^2)}$. The spherical zone swept from the equator ($\theta = \pi/2$) to the latitude of the leading edge (θ_N) has area given by Archimedes' theorem: the area of a spherical zone equals the area of the circumscribed cylinder of the same height. Zone height $h_N = F\cdot\cos(\pi/2) - F\cdot\cos(\theta_N)$ evaluated from the equator going north: $h_N = F \times N\delta/\sqrt{(F^2 + N^2\delta^2)}$.

EQUATION SET 11 — THE ZONE AREA FORMULA

$$A_{zone}(N) = 2\pi F \times h_N = 2\pi F^2 \times N\delta/\sqrt{(F^2+N^2\delta^2)} \quad (1)$$
└ Archimedes zone area = 2π × sphere radius × zone height

$$F^2 = 9/25,\ \delta = 1/10:\ F^2 + N^2\delta^2 = 9/25 + N^2/100 = (36+N^2)/100 \quad (2)$$
└ simplify denominator with Hardlock values

$$A_{zone}(N) = 2\pi(9/25) \times N(1/10)/\sqrt{((36+N^2)/100)} \quad (3)$$
└ substituting Hardlock values

$$= (18\pi N/250) \times 10/\sqrt{(36+N^2)} = 18\pi N/(25\sqrt{(36+N^2)}) \quad (4)$$
└ ZONE AREA FORMULA — LOCKED

$$(F/\delta)^2 = (0.6/0.1)^2 = 36:\ \text{factor } 36 = (F/\delta)^2 \text{ in denominator} \quad (5)$$
└ the sphere-to-lattice ratio squared appears explicitly

The factor $36 = (F/\delta)^2 = 6^2$ in the denominator encodes the ratio of the sphere radius to the lattice grain. This ratio — six lattice steps per sphere radius — was derived in Section 5 (Equation Set 3). The zone area formula $A_{zone}(N) = 18\pi N/(25\sqrt{(36+N^2)})$ is an exact closed-form result with zero free parameters.

10.3 The Coverage Fraction

The coverage fraction is A_zone(N) divided by the total sphere area $4\pi F^2$:

EQUATION SET 12 — THE COVERAGE FRACTION

$$f(N) = A_{zone}(N) / (4\pi F^2) \quad (1)$$
└ *coverage fraction: fraction of $S^2(0.6)$ swept after N Chronons*

$$4\pi F^2 = 4\pi(9/25) = 36\pi/25 \quad (2)$$
└ *total sphere area — LOCKED*

$$f(N) = [18\pi N/(25\sqrt{(36+N^2)})] / [36\pi/25] \quad (3)$$
└ *substituting both expressions*

$$= [18\pi N/(25\sqrt{(36+N^2)})] \times [25/(36\pi)] = 18N / (36\sqrt{(36+N^2)}) \quad (4)$$
└ *simplification*

$$f(N) = N / (2\sqrt{(36+N^2)}) \quad (5)$$
└ *COVERAGE FRACTION — LOCKED*

$$\lim(N\to\infty)\, f(N) = \lim N/(2N\sqrt{(1+36/N^2)}) = 1/2 \quad (6)$$
└ *northern hemisphere asymptote: coverage approaches 50%*

The coverage fraction $f(N) = N/(2\sqrt{(36+N^2)})$ depends on N and the ratio $F/\delta = 6$ (both locked). As $N \to \infty$, $f(N) \to 1/2$: the coverage fraction approaches exactly 50%. This asymptote is structural: the worldline starts at the equator (z=0) and advances only northward ($z > 0$), so the maximum coverage is the northern hemisphere (50%). The southern hemisphere (50%) is permanently inaccessible to the worldline's projection from z=0 going north.

10.4 Coverage After N_sat = 25 Chronons

EQUATION SET 13 — COVERAGE AFTER N_sat = 25 CHRONONS

$$N_{sat} = 25 \quad \text{(Essay II, T-5.2: } N_{sat} = 1/\rho = 1/0.04 = 25\text{)} \tag{1}$$
└ *Saturation Threshold — LOCKED*

$$f(25) = 25 / (2\sqrt{(36 + 625)}) = 25 / (2\sqrt{661}) \tag{2}$$
└ *$36 + N_{sat}^2 = 36 + 625 = 661$*

$$\sqrt{661} \approx 25.71;\ f(25) \approx 25/(2\times25.71) \approx 25/51.42 \approx 0.4862 \approx 48.6\% \tag{3}$$
└ *coverage fraction after N_sat Chronons*

$$A_{zone}(25) = 18\pi\times25 / (25\sqrt{661}) = 18\pi/\sqrt{661} \approx 2.20 \text{ lattice}^2 \tag{4}$$
└ *zone area after 25 Chronons*

$$\text{Latent fraction: } 1 - f(25) = (2\sqrt{661}-25)/(2\sqrt{661}) \approx 51.4\% \tag{5}$$
└ *residual latent capacity after N_sat Chronons*

THEOREM — T-10.1 — The Coverage Fraction After N_sat Chronons

After $N_{sat} = 25$ Chronons, the exact cumulative coverage fraction of $S^2(0.6)$ is:

$f(N_{sat}) = 25/(2\sqrt{661}) \approx 48.6\%$.

The latent fraction is $1 - f(25) = (2\sqrt{661} - 25)/(2\sqrt{661}) \approx 51.4\%$.

Distinction from the per-Chronon beam: the per-Chronon beam activation $12.38\% = \Omega/(4\pi) = 14/(36\pi)$ is the INSTANTANEOUS fraction of the sphere illuminated by the helicon's cross-section at any single Chronon. The 48.6% is the CUMULATIVE non-overlapping coverage (the distinct fraction of the sphere the worldline's projection has swept) after all $N_{sat} = 25$ Chronons. □

10.5 The Beam Solid Angle vs Cumulative Coverage — Reconciliation

The relationship between the per-Chronon beam activation (12.38%) and the cumulative coverage after N Chronons (f(N)) deserves explicit reconciliation. If consecutive beam spots never overlapped, cumulative coverage after N Chronons would be N × 12.38%. After N = 25: 25 × 12.38% = 309.5%, which is impossible. The ratio 309.5% / 48.6% ≈ 6.37 represents the average number of times each covered surface element is illuminated. Consecutive beam positions overlap almost completely.

The physical reason: the beam solid angle $\Omega = 14/9$ sr subtends a cone of half-angle α_{beam} where $2\pi(1 - \cos(\alpha_{beam})) = 14/9$, giving $\cos(\alpha_{beam}) = 1 - 7/(9\pi) \approx 0.752$, $\alpha_{beam} \approx 41.2°$. Each Chronon's beam covers a cone of half-angle 41.2°. The loxodrome advances in latitude by approximately $\alpha \approx 3.07°$ per Chronon. Since $3.07° \ll 41.2°$, consecutive beams overlap by approximately $(41.2 - 3.07)/41.2 \approx 92.5\%$ of their area. The new territory illuminated per Chronon is only about 7.5% of the beam area, growing the distinct coverage by $12.38\% \times 7.5\% \approx 0.93\%$ per Chronon rather than 12.38%.

The cumulative coverage formula $f(N) = N/(2\sqrt{(36+N^2)})$ captures this overlap exactly. It is not an approximation: it computes the exact zone area swept by the loxodrome, which accounts for the non-overlapping advance of the worldline's leading edge. The formula grows slowly precisely because the loxodrome advances in latitude at pitch angle $\alpha = 3.07°$ per Chronon, far less than the beam half-angle of 41.2°.

COROLLARY — C-10.1 — The 50% Northern Hemisphere Asymptote

The exact cumulative coverage formula $f(N) = N/(2\sqrt{(36+N^2)})$ has:

$\lim(N\to\infty) f(N) = 1/2$.

The worldline can cover at most 50% of $S^2(0.6)$ because it starts at the equator and advances only northward ($z > 0$ for all $\tau > 0$). The northern hemisphere (50%) is the maximum achievable coverage. The southern hemisphere (50%) remains permanently latent.

After $N_{sat} = 25$ Chronons: coverage = 48.6%. The system has used 48.6% of its northern hemisphere registration capacity. The remaining 1.4% of the northern hemisphere and the full 50% southern hemisphere (total latent = 51.4%) constitute the structural reservoir for Phase III Level n=3 self-registration. □

11. G-4 Closure and Anticipation of Essay VII

The derivations of Sections 2 through 10 constitute the complete closure of Gap G-4. Every geometric constant of the helical worldline is a zero-free-parameter consequence of the primitive suite. The following table and notice constitute the formal closure statement.

Table 1. Complete Derivational Register — Essay VI Contributions

Symbol	Value	Exact Form	Derivational Path	Status
$\gamma(\tau)$	$(F\cdot\cos(\omega\tau), F\cdot\sin(\omega\tau), c\tau)$	Exact	Three-component structural derivation Sections 3–6	LOCKED
ω_{pc}	28/9 rad/Chronon	$2EC/F^2 = 2\Omega$	Area sweep condition: sector = A_EC (Section 4)	LOCKED
v_{pc}	δ = 0.1/Chronon	$c\tau_0 = \delta$	Speed of Causality × Chronon duration (Section 5)	LOCKED
$\tan(\alpha)$	3/56 ≈ 0.0536	$k/(F\omega_{pc}\times 30)$	$v_{pc}/(F\omega_{pc})$ = (1/10)/(28/15) (Section 7)	LOCKED
L_{pc}	$\sqrt{3145}/30 \approx 1.869$	$\sqrt{(56^2+k^2)}/30$	Pythagoras: $(F\omega_{pc})^2+(v_{pc})^2$ (Section 7)	LOCKED
p	$9\pi/140 \approx 0.202$	$\delta\times 9\pi/14$	$\delta \times 2\pi/\omega_{pc} = \delta \times 9\pi/14$ (Section 7)	LOCKED
Win./Chronon	$14/(9\pi) \approx 0.495$	Ω/π	$\omega_{pc}/(2\pi)$ = (28/9)/(2π) (Section 7)	LOCKED
Ω_{exact}	$(14/9)(56/\sqrt{3145})$	$\Omega\cdot\cos(\alpha)$	$\Omega_{planar}\times\cos(\alpha)$; correction 0.144% (Section 9)	LOCKED
$A_{zone}(N)$	$18\pi N/(25\sqrt{(36+N^2)})$	Exact	Archimedes zone: $2\pi F^2 \times h_N$ (Section 10)	LOCKED
f(N)	$N/(2\sqrt{(36+N^2)})$	$A_{zone}/(4\pi F^2)$	Coverage fraction, asymptote 1/2 (Section 10)	LOCKED
f(25)	$25/(2\sqrt{661}) \approx 48.6\%$	Exact	$f(N_{sat})$: 36+625=661 (Section 10, T-10.1)	LOCKED
Asymptote	$\lim f(N)=1/2$	50%	Northern hemisphere: worldline starts equator, advances north (C-10.1)	LOCKED

NOTICE — N-11.1 — G-4 Formal Closure

Gap G-4 is hereby formally closed. The derivational chain from the primitive suite to the complete helical worldline and its integrated arc coverage is:

1. d=3 uniqueness: Intersection Crisis forecloses d≤2 (Jordan Curve Theorem + ε_snap arithmetic); thermodynamic sink forecloses d>3 (Section 2).
2. Three-component worldline: E→cosine, C→sine, F→axial; r=F=0.6; v=c=δ/τ₀ (Sections 3–5).
3. Angular frequency: $\omega_{pc} = 2EC/F^2 = 2\Omega = 28/9$ rad/Chronon from area sweep condition (Section 4).
4. Helical parameters: $\tan(\alpha)=3/56$; $L_{pc}=\sqrt{3145}/30$; $p=9\pi/140$; windings/Chronon=Ω/π (Section 7).
5. Self-avoiding: strict z-monotonicity; $z(n)=n\delta$ uniquely indexes every Chronon (Section 8).
6. Exact solid angle: $\Omega_{exact} = (14/9)(56/\sqrt{3145})$; correction 0.144% from planar (Section 9).
7. Zone area: $A_{zone}(N)=18\pi N/(25\sqrt{(36+N^2)})$; $f(N)=N/(2\sqrt{(36+N^2)})\to 1/2$ (Section 10).
8. At $N_{sat}=25$: $f(25)=25/(2\sqrt{661})\approx 48.6\%$; latent reserve 51.4% (T-10.1, C-10.1).

All entries LOCKED. G-4 CLOSED.

11.2 Anticipation of Essay VII — N_sat from Sphere Resolution

Essay VII will derive $N_{sat} = 25$ from the sphere's discrete resolution limit: the maximum number of distinct non-overlapping registration events that can be inscribed on $S^2(0.6)$ at δ-lattice resolution before the coverage saturates. This is a second, independent derivation of $N_{sat} = 25$ — distinct from Essay II's derivation via $N_{sat} = 1/\rho = 1/0.04 = 25$.

The exact coverage formula $f(N) = N/(2\sqrt{(36+N^2)})$ provides the geometric foundation for this derivation. The saturation condition: $f(N_{sat})$ must equal the sphere's resolution saturation fraction — the point at which the loxodrome can no longer inscribe new territory at δ-resolution. Essay VII will show that this condition determines $N_{sat} = 25$ geometrically, coinciding with the informational derivation of Essay II. Two independent derivations converge on the same value: $N_{sat} = 25 = 1/\rho$ (informational) = $F/\delta \times k$ (geometric, $F/\delta = 6$, $k = k_{minimal}$). This convergence is not a coincidence but a structural consequence of the Triadic Architecture. The present essay establishes the geometric foundation required: $\gamma(\tau)$, ω_{pc}, the zone area formula, and $f(N)$.

Conclusion

This essay has closed Gap G-4 — the exact derivation of the kinetic worldline and its integrated arc coverage on $S^2(0.6)$ — through nine staged derivational closures, each building directly on the locked results of Essays I through V without introducing a free parameter.

The Intersection Crisis (Section 2) established why $d \leq 2$ is foreclosed and $d=3$ is the unique dimensionality for a self-avoiding worldline. The Jordan Curve Theorem forces self-intersection in $d=2$; the absence of two-dimensional area in $d=1$ violates the Triadic Mandate; and the F-axis in $d=3$ provides the over-under clearance that absorbs the resolution remainder $3\varepsilon_{snap} = \delta$ without collision. The $d=3$ foreclosure is topological, not just combinatorial.

The worldline decomposition (Sections 3–5) derived the three structurally independent components: E→cosine (phase-leading generative drive), C→sine (90°-lagging limitative constraint), F→linear axial advance (Registration as the propagation axis). The radius $r = F = 0.6$ is the Registration boundary, shared by all rotational components because the worldline must remain above the discriminability floor at all times.

The angular frequency (Section 4) was derived from the area sweep condition: $\omega_{pc} = 2EC/F^2 = 2\Omega = 28/9$ rad/Chronon, exactly twice the solid angle per Chronon. The axial velocity (Section 5) was derived as $v_{pc} = c\tau_0 = \delta$, the exact axial advance coupling the worldline to the $d=3$ clearance mechanism and the Fundamental Stable Knot $k=3$.

The helical parameters (Section 7) follow with zero free parameters. The pitch angle $\tan(\alpha) = 3/56$ exhibits the Fundamental Stable Knot $k=3$ as its numerator. The arc length $L_{pc} = \sqrt{3145}/30 = \sqrt{(56^2 + k^2)}/30$ exhibits k as the axial term of the Pythagorean decomposition. The helical pitch $p = 9\pi/140$ produces approximately 5 tight windings per lattice unit of axial advance.

The self-avoiding property (Section 8) is proven by strict z-monotonicity: $z(n) = n\delta$ uniquely indexes every Chronon, and the worldline is injective by virtue of $c \neq 0$. The exact solid angle correction (Section 9) is $\Omega_{exact} = (14/9)(56/\sqrt{3145})$, a correction of 0.144% from Essay III's planar approximation, confirming the approximation was structurally justified.

The zone area formula (Section 10) gives the exact cumulative coverage: $A_{zone}(N) = 18\pi N/(25\sqrt{(36+N^2)})$, yielding coverage fraction $f(N) = N/(2\sqrt{(36+N^2)})$ with asymptote 1/2. After $N_{sat} = 25$ Chronons: $f(25) = 25/(2\sqrt{661}) \approx 48.6\%$, approaching the 50% northern hemisphere limit.

The distinction between instantaneous beam activation (12.38% per Chronon) and cumulative distinct coverage (48.6% after 25 Chronons) is resolved: the beam half-angle of 41.2° vastly exceeds the per-Chronon latitude advance of 3.07°, so consecutive beams overlap by approximately 92.5%. The zone formula captures this overlap exactly.

PRINCIPLE — Essay Contribution Summary

Gap(s) closed: G-4 — Exact worldline $\gamma(\tau)$; all helical parameters from Hardlock; exact arc coverage replacing planar approximation

Core results: $\omega_{pc}=2\Omega=28/9$; $\tan(\alpha)=3/56$; $L_{pc}=\sqrt{3145}/30$; $p=9\pi/140$; windings/Chronon$=\Omega/\pi$; $\Omega_{exact}=(14/9)(56/\sqrt{3145})$; $f(N)=N/(2\sqrt{(36+N^2)})$; $f(25)=25/(2\sqrt{661})\approx 48.6\%$; asymptote=50%

Structural identities: $\omega_{pc}=2\Omega$; windings/Chronon$=\Omega/\pi$; $\tan(\alpha)=k/56$ where k=Fundamental Stable Knot=3; $L_{pc}=\sqrt{(56^2+k^2)}/30$

Derivational status: CLOSED — G-4 formally closed; all helical constants zero-free-parameter; Essay VII proceeds to the Saturation Equivalence on the basis of the locked zone formula $f(N)=N/(2\sqrt{(36+N^2)})$

Epistemic classification: Derivationally saturated. Kinetic worldline geometry complete. Phase III geometric reservoir quantified at 51.4% after N_{sat} Chronons.

References

All internal framework derivations are sourced from the Gradientology Framework (Pretorius, 2026). External references follow APA 7th edition.

Archimedes. (c. 225 BCE). On the Sphere and Cylinder (Book I, Proposition 42–43). [1]

Jordan, Camille. (1887). Cours d'analyse de l'École Polytechnique (Vol. 3). Paris: Gauthier-Villars.[2]

Mercator, Gerardus. (1569). Nova et Aucta Orbis Terrae Descriptio ad Usum Navigantium Emendate Accommodata. Duisburg: Rumold Mercator (published posthumously in the 1595 atlas)[3]

Note on usage

All internal framework derivations are sourced from the Gradientology Framework (Pretorius, 2026). The external entries above are listed as structural isomorphisms (confirmatory, not foundational) — they name parallel ideas but are not logical sources for any derivation

Structural Isomorphisms (confirmatory, not foundational): Jordan, C. (1887) on the Jordan Curve Theorem; Archimedes (c. 250 BCE) on the spherical zone area theorem (equal to the circumscribed cylinder of the same height); Mercator, G. (1569) on the loxodrome and constant-bearing curves on spheres. These independently name the same structural necessities the helical worldline produces internally. They are not sources.

Internal series references

Pretorius, E. (2026). *Gradientology - Foundations of the Primordial Triad*: Treatises I-XVI. Zenodo.

Pretorius, E. (2026). *Gradient Mechanics - The Dynamics of the Inversion Principle*: Corpus Papers I–XVI. Zenodo.

Pretorius, E. (2026). *Gradient Cybernetics: The Calculus of Recursion*: Synthesis Essays I–XVI. Zenodo.

[1] Modern edition: Heath, Thomas L. (Ed. & Trans.). (1897). The Works of Archimedes. Cambridge: Cambridge University Press, pp. 91–98

[2] The theorem appears in the 1887 edition as part of the topological treatment of simple closed curves in the plane.

[3] The 1569 world map introduces the projection in which rhumb lines (constant-bearing courses) appear as straight lines; the mathematical property of the loxodrome is implicit in the projection's construction

Addendum A — Terms of Engagement: Recursive Interpretive Constraints

The following Recursive Interpretive Constraints are codified as binding Terms of Engagement for all sixteen essays in this series.

A.1 Substrate Neutrality

PRINCIPLE — A.1 — Substrate Neutrality

No reader may attribute consciousness, mind, or sentience to a specific biological substrate. These are derived as Phase Transitions of Coherence occurring when integrated information density exceeds the Coherence Threshold. Whether the substrate is biological, silicon, or stellar, the emergence of a Subject is the structural mandate of the Inversion Principle achieving a unified inside view.

A.2 Foreclosure of the Ego as Substance

THEOREM — A.2 — The Ego as Strange Attractor

The Self is derived as a Strange Attractor in neural phase space — a mathematical singularity when the registration function becomes recursive (F→F). It is a Topological Knot, not an ontological addition.

A.3 Intelligence as Virtualisation

DEFINITION — A.3 — Intelligence

Intelligence is the Virtualisation of the Evolutionary Algorithm: the capacity of F to operate on Internal Flux (counterfactual simulation), allowing hypotheses to die in simulation so the organism persists.

A.4 Agency as Ontological Gap Work

DERIVATION — A.4 — Agency as Ontological Gap Work

Agency and Free Will are the mechanical consequences of the Ontological Gap ($1-m \approx 0.298$). Agency is Top-Down Modulation of Constraints (C) to steer generative flux (E) through calculated indeterminacy.

A.5 Memory as Recursive Depth

DEFINITION — A.5 — Memory

Memory is localised recursive depth (k): the number of update cycles required to resolve a local state to within lattice tolerance. It is a Processing Load Artefact.

A.6 Adaptation as Inversion Optimisation

PRINCIPLE — A.6 — Adaptation and Prediction

Adaptation is the optimisation of the Inversion Principle's throughput against tightening registration grain.

A.7 Re-coding Emergence

THEOREM — A.7 — Substrate Saturation $N_{sat}=25$

All higher-order cognitive capacities are re-coded as Substrate Saturation ($N_{sat}=25$). Mandatory Coarse-Graining begins when recursive state density reaches saturation.

A.8 Final Demarcation Statement

Phase III establishes that the universe is not a collection of things but a scalar-invariant calculation of its own existence. Any attempt to treat these derivations as anthropocentric management tools is formally identified as Logical Necrosis.

NOTICE — A.8 — Binding Constraint on All Readers

These Terms of Engagement are non-negotiable and apply universally to all sixteen essays. Violation constitutes a formal epistemic breach classified as Logical Necrosis within the Gradientology.

Addendum B — Derivational Ledger

The following ledger enumerates every Principle, Definition, Theorem, Derivation, Corollary, and Notice in this essay.

Table B1. Complete Derivational Ledger — Essay VI

Type	Reference Label	Essay	Status
THEOREM	T-2.1 — The Intersection Crisis in d=2	VI	LOCKED
THEOREM	T-2.2 — d=3 Is the Unique Dimensionality Resolving the Intersection Crisis	VI	LOCKED
DERIVATION	D-2.1 — The d=1 Foreclosure	VI	LOCKED
PRINCIPLE	P-3.1 — Structural Derivation of the Three Worldline Components	VI	LOCKED
THEOREM	T-4.1 — The Angular Frequency Is Unique	VI	LOCKED
COROLLARY	C-5.1 — The Axial Advance Enables d=3 Clearance	VI	LOCKED
DERIVATION	D-6.1 — Foreclosure of the Closed Circular Orbit ($v_{pc}=0$)	VI	LOCKED
DERIVATION	D-6.2 — Foreclosure of the Straight-Line Path ($\omega_{pc}=0$)	VI	LOCKED
THEOREM	T-8.1 — The Worldline Is Strictly Self-Avoiding	VI	LOCKED
COROLLARY	C-8.1 — Lattice Self-Avoidance of the Worldline	VI	LOCKED
COROLLARY	C-9.1 — The Planar Approximation Is Accurate to 0.15%	VI	LOCKED
THEOREM	T-10.1 — The Coverage Fraction After N_{sat} Chronons	VI	LOCKED
COROLLARY	C-10.1 — The 50% Northern Hemisphere Asymptote	VI	LOCKED
NOTICE	N-11.1 — G-4 Formal Closure	VI	LOCKED
PRINCIPLE	Essay Contribution Summary	VI	LOCKED

Addendum C — Full Derivational Trace — Primitives to Worldline and Coverage

Complete step-by-step trace from the Hardlock primitive suite to the helical worldline and its integrated arc coverage.

Table C1. Full Derivational Trace — G-4 Closure

Step	Type	Input	Operation / Result
1	T	d=2, JCT, ε_snap	Intersection Crisis: d≤2 forces self-intersection; d=3 unique resolution (T-2.1, T-2.2)
2	L	E,C,F roles	Components: E→cos(ωτ), C→sin(ωτ), F→cτ; radius r=F=0.6 (P-3.1)
3	A	A_EC=EC=14/25, F^2=9/25	ω_pc = 2EC/F^2 = 2Ω = 28/9 rad/Chronon (T-4.1, Eq.Set 1)
4	I	c=δ/τ₀=0.01	v_pc = cτ₀ = δ = 0.1/Chronon; F=6δ (Eq.Set 3, C-5.1)
5	I	ω_pc, v_pc, r=F	γ(τ)=(F·cos(ωτ), F·sin(ωτ), cτ); circle and line foreclosed (D-6.1, D-6.2)
6	A	v_pc/(Fω_pc)	tan(α) = 3/56 exactly; α ≈ 3.07°; numerator = k = 3 (Eq.Set 5)
7	A	(Fω_pc)² + v_pc²	L_pc = √3145/30 = √(56²+k²)/30; p=9π/140; windings=Ω/π (Eq.Sets 6,7,8)
8	T	c>0, z(τ)=cτ	γ injective: γ(τ₁)≠γ(τ₂) for τ₁≠τ₂; lattice z(n)=nδ unique (T-8.1, C-8.1)
9	A	Ω_planar, cos(α)	Ω_exact=(14/9)(56/√3145); correction 0.144% (Section 9, C-9.1)
10	A	2πF^2, Nδ, F^2+$N^2δ^2$	A_zone(N)=18πN/(25√(36+N^2)); f(N)=N/(2√(36+N^2)) (Eq.Sets 11,12)
11	A	N_sat=25	f(25)=25/(2√661)≈48.6%; asymptote=50% (T-10.1, C-10.1)
12	L	All above	G-4 CLOSED; foundation for Essay VII Saturation Equivalence confirmed (N-11.1)

Addendum D — Open Derivational Gaps — Status After Essay VI

Status of all open derivational gaps after the closure of G-4.

G-1: CLOSED by Essay II. Informational ground complete. All LOCKED.

G-2: CLOSED by Essay III. C^3, $S^2(0.6)$, r=F, $\Omega=14/9$. All LOCKED.

G-3a: CLOSED by Essay IV. $F^*=0.6$ fixed point, r(G)=1, k=3, Non-Equilibrium Theorem. All LOCKED.

G-3b: CLOSED by Essay V. Regulatory Monopoly: G=EC/F unique among all nine algebraic forms. All LOCKED.

G-4: CLOSED by Essay VI. Exact worldline $\gamma(\tau)$; $\omega_{pc}=28/9$; $\tan(\alpha)=3/56$; $L_{pc}=\sqrt{3145}/30$; $p=9\pi/140$; $f(N)=N/(2\sqrt{(36+N^2)})$; $f(25)=25/(2\sqrt{661})$. All LOCKED.

G-5 (Essay VII): $N_{sat}=25$ from spherical resolution — equivalence between informational derivation (Essay II: $N_{sat}=1/\rho=25$) and sphere geometry (Essay VI zone formula). Two independent derivations converging on $N_{sat}=25$. Closed by Essay VII.